OCR RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Given that $y = e^{-x}\sin 2x$, find $\frac{dy}{dx}$. **[3]**

(ii) Hence show that the curve $y = e^{-x}\sin 2x$ has a stationary point when $x = \frac{1}{2}\arctan 2$. **[3]**

2 A curve has equation $x^2 + 2y^2 = 4x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] **[3]**

3 Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined. **[2]**

4 The temperature θ °C of water in a container after t minutes is modelled by the equation

$$\theta = a - be^{-kt},$$

where a, b and k are positive constants.

The initial and long-term temperatures of the water are 15 °C and 100 °C respectively. After 1 minute, the temperature is 30 °C.

(i) Find a, b and k. **[6]**

(ii) Find how long it takes for the temperature to reach 80 °C. **[2]**

5 The driving force F newtons and velocity v km s^{-1} of a car at time t seconds are related by the equation $F = \frac{25}{v}$.

(i) Find $\frac{dF}{dv}$. **[2]**

(ii) Find $\frac{dF}{dt}$ when $v = 50$ and $\frac{dv}{dt} = 1.5$. **[3]**

6 Evaluate $\int_0^3 x(x+1)^{-\frac{1}{2}}\,dx$, giving your answer as an exact fraction. **[5]**

7 **(i)** Disprove the following statement:

$$3^n + 2 \text{ is prime for all integers } n \geqslant 0.$$ **[2]**

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. **[2]**

Section B (36 marks)

8 Fig. 8 shows parts of the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, where $\mathrm{f}(x) = \tan x$ and $\mathrm{g}(x) = 1 + \mathrm{f}(x - \frac{1}{4}\pi)$.

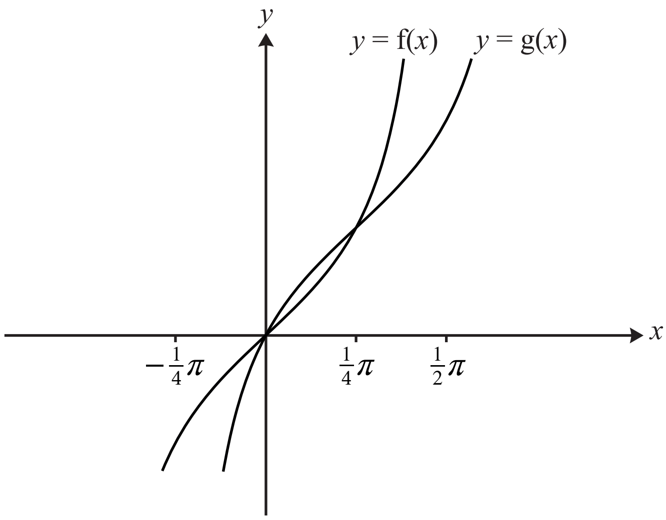

Fig. 8

(i) Describe a sequence of two transformations which maps the curve $y = \mathrm{f}(x)$ to the curve $y = \mathrm{g}(x)$. **[4]**

It can be shown that $\mathrm{g}(x) = \dfrac{2\sin x}{\sin x + \cos x}$.

(ii) Show that $\mathrm{g}'(x) = \dfrac{2}{(\sin x + \cos x)^2}$. Hence verify that the gradient of $y = \mathrm{g}(x)$ at the point $(\frac{1}{4}\pi, 1)$ is the same as that of $y = \mathrm{f}(x)$ at the origin. **[7]**

(iii) By writing $\tan x = \dfrac{\sin x}{\cos x}$ and using the substitution $u = \cos x$, show that $\displaystyle\int_0^{\frac{1}{4}\pi} \mathrm{f}(x)\,\mathrm{d}x = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{u}\,\mathrm{d}u$.
Evaluate this integral exactly. **[4]**

(iv) Hence find the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$, the x-axis and the lines $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$. **[2]**

9 Fig. 9 shows the line $y = x$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x - 1)$. The line and the curve intersect at the origin and at the point P(a, a).

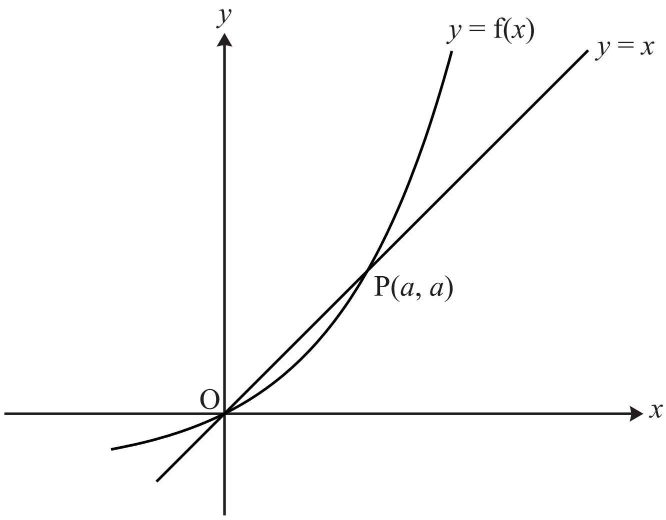

Fig. 9

(i) Show that $\mathrm{e}^a = 1 + 2a$. **[1]**

(ii) Show that the area of the region enclosed by the curve, the x-axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a, the area enclosed by the curve and the line $y = x$. **[6]**

(iii) Show that the inverse function of $\mathrm{f}(x)$ is $\mathrm{g}(x)$, where $\mathrm{g}(x) = \ln(1 + 2x)$. Add a sketch of $y = \mathrm{g}(x)$ to the copy of Fig. 9. **[5]**

(iv) Find the derivatives of $\mathrm{f}(x)$ and $\mathrm{g}(x)$. Hence verify that $\mathrm{g}'(a) = \dfrac{1}{\mathrm{f}'(a)}$.

Give a geometrical interpretation of this result. **[7]**

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

Section A (36 marks)

1 (i)	
1 (ii)	

2 (i)	
2 (ii)	

3	

4 (i)	

4 (ii)	

5 (i)	
5 (ii)	

6

7 (i)	
7 (ii)	

Section B (36 marks)

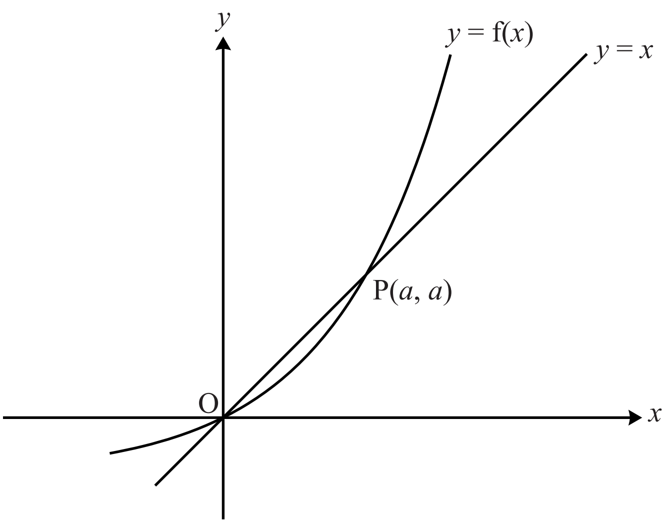
8 (i)	
8 (ii)	
	(answer space continued on next page)

8 (ii)	(continued)

8 (iii)	
8 (iv)	

9 (i)	
9 (ii)	

9 (iii)



9 (iv)	
	(answer space continued on next page)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for January 2013

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$y = e^{-x} \sin 2x$ $\Rightarrow$ $dy/dx = e^{-x}.2\cos 2x + (-e^{-x})\sin 2x$	M1 B1 A1 **[3]**	Product rule $d/dx(\sin 2x) = 2\cos 2x$ Any correct expression	$u \times$ their $v' + v \times$ their u' but mark final answer
1	(ii)		$dy/dx = 0$ when $2\cos 2x - \sin 2x = 0$ $\Rightarrow$ $2 = \tan 2x$ $\Rightarrow$ $2x = \arctan 2$ $\Rightarrow$ $x = ½ \arctan 2$ *	M1 M1 A1 **[3]**	ft their dy/dx but must eliminate e^{-x} $\sin 2x / \cos 2x = \tan 2x$ used [or $\tan^{-1}$] **NB AG**	derivative must have 2 terms substituting ½ arctan 2 into their deriv M0 (unless $\cos 2x = 1/\sqrt{5}$ and $\sin 2x = 2/\sqrt{5}$ found) must show previous step
2	(i)		$2x + 4y\frac{dy}{dx} = 4$ $\Rightarrow$ $\frac{dy}{dx} = \frac{4-2x}{4y}$	M1 A1 A1 **[3]**	$4y\frac{dy}{dx}$ correct equation o.e., but mark final answer	Rearranging for y and differentiating explicitly is M0 Ignore superfluous $dy/dx = \ldots$ unless used subsequently
2	(ii)		$\frac{dy}{dx} = 0 \Rightarrow x = 2$ $\Rightarrow$ $4 + 2y^2 = 8 \Rightarrow y^2 = 2, y = \sqrt{2}$ or $-\sqrt{2}$	B1dep B1B1 **[3]**	dep correct derivative $\sqrt{2}, -\sqrt{2}$	 can isw, penalise inexact answers of ±1.41 or better once only −1 for extra solutions found from using $y = 0$
3			$1 < x < 3 \Rightarrow$ $-1 < x - 2 < 1$ $\Rightarrow$ $\lvert x-2 \rvert < 1$	 B1 B1 **[2]**	oe [or $a = 2$ and $b = 1$]	

Question			Answer	Marks	Guidance	
4	(i)		$\theta = a - be^{-kt}$			
			When $t = 0$, $\theta = 15 \Rightarrow 15 = a - b$	M1	$15 = a - b$	must have $e^0 = 1$
			When $t = \infty$, $\theta = 100 \Rightarrow 100 = a$	B1	$a = 100$	
			$\Rightarrow \quad b = 85$	A1cao	$b = 85$	
			When $t = 1$, $\theta = 30 \Rightarrow 30 = 100 - 85e^{-k}$	M1	$30 = a - b\,e^{-k}$	(need not substitute for a and b)
			$\Rightarrow \quad e^{-k} = 70/85$			
			$\Rightarrow \quad -k = \ln(70/85) = -0.194(156\ldots)$	M1	Re-arranging and taking lns	allow $-k = \ln[(a - 30)/b]$ ft on a, b
			$\Rightarrow \quad k = 0.194$	A1	0.19 or better, or $-\ln(70/85)$ oe	mark final ans
				[6]		
4	(ii)		$80 = 100 - 85\,e^{-0.194t}$	M1	ft their values for a, b and k	but must substitute values
			$\Rightarrow \quad e^{-0.194t} = 20/85$			
			$\Rightarrow \quad t = -\ln(4/17) / 0.194 = 7.45$ (min)	A1	art 7.5 or 7 min 30 s or better	
				[2]		
5	(i)		$dF/dv = -25\,v^{-2}$	M1	$d/dv(v^{-1}) = -v^{-2}$ soi	
				A1	$-25\,v^{-2}$ o.e mark final ans	
				[2]		
5	(ii)		When $v = 50$, $dF/dv = -25/50^2$ (= -0.01)	B1	$-25/50^2$	
			$\frac{dF}{dt} = \frac{dF}{dv}.\frac{dv}{dt}$	M1	o.e.	e.g. $\frac{dF}{dv} = \frac{dF}{dt} / \frac{dv}{dt}$
			$= -0.01 \times 1.5 = -0.015$	A1cao	o.e. e.g. $-3/200$ isw	
				[3]		

Question			Answer	Marks	Guidance	
6			Let $u = 1 + x \Rightarrow$ $\int_0^3 x(1+x)^{-1/2}\,dx = \int_1^4 (u-1)u^{-1/2}\,du$	M1	$\int (u-1)u^{-1/2}(du)$ *	condone no du, missing bracket, ignore limits
			$= \int_1^4 (u^{1/2} - u^{-1/2})\,du$	A1	$\int (u^{1/2} - u^{-1/2})(du)$	
			$= \left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^4$	A1	$\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]$ o.e.	e.g. $\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right]$; ignore limits
			= (16/3 – 4) – (2/3 – 2)	M1dep	upper–lower dep 1st M1 and integration	with correct limits e.g. 1, 4 for u or 0, 3 for x
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	or using $w = (1+x)^{1/2} \Rightarrow$ $\int \frac{(w^2-1)2w}{w}(dw)$ M1
			OR Let $u = x$, $v' = (1+x)^{-1/2}$	M1		$= \int 2(w^2-1)(dw)$ A1 $= \left[\frac{2}{3}w^3 - 2w\right]$ A1
			$\Rightarrow$ $u' = 1$, $v = 2(1+x)^{1/2}$	A1		upper–lower with correct limits (w = 1,2) M1
			$\Rightarrow$ $\int_0^3 x(1+x)^{-1/2}\,dx = \left[2x(1+x)^{1/2}\right]_0^3 - \int_0^3 2(1+x)^{1/2}\,dx$	A1	ignore limits, condone no dx	8/3 A1 cao
			$= \left[2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2}\right]_0^3$	A1	ignore limits	*If $\int_1^4 (u-1)u^{-1/2}\,du$ done by parts:
			$= (2 \times 3 \times 2 - 4 \times 8/3) - (0 - 4/3)$			$2u^{1/2}(u-1) - \int 2u^{1/2}\,du$ A1
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	$[2u^{1/2}(u-1) - 4u^{3/2}/3]$ A1 substituting correct limits M1 8/3 A1cao
				[5]		

Question			Answer	Marks	Guidance	
7	(i)		$3^5 + 2 = 245$ [which is not prime]	M1 A1 **[2]**	Attempt to find counter-example correct counter-example identified	If A0, allow M1 for $3^n + 2$ correctly evaluated for 3 values of n
7	(ii)		$(3^0 = 1)$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, … so units digits cycle through 1, 3, 9, 7, 1, 3, … so cannot be a '5'. **OR** 3^n is not divisible by 5 all numbers ending in '5' are divisible by 5. so its last digit cannot be a '5'	M1 A1 B1 B1 **[2]**	Evaluate 3^n for n = 0 to 4 or 1 to 5 must state conclusion for B2	allow just final digit written
8	(i)		translation in the x–direction of $\pi/4$ to the right translation in y-direction of 1 unit up.	M1 A1 M1 A1 **[4]**	allow 'shift', 'move' oe (eg using vector) allow 'shift', 'move' oe (eg using vector)	If just vectors given withhold **one** 'A' mark only 'Translate $\begin{pmatrix} \pi/4 \\ 1 \end{pmatrix}$' is 4 marks; if this is followed by an additional incorrect transformation, SC M1M1A1A0 $\begin{pmatrix} \pi/4 \\ 1 \end{pmatrix}$ only is M2A1A0

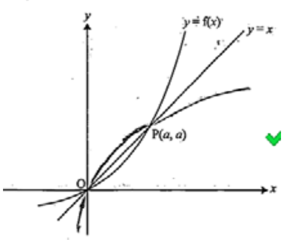
Question			Answer	Marks	Guidance	
8	(ii)		$g(x)=\dfrac{2\sin x}{\sin x+\cos x}$			(Can deal with num and denom separately)
			$g'(x)=\dfrac{(\sin x+\cos x)2\cos x-2\sin x(\cos x-\sin x)}{(\sin x+\cos x)^2}$	M1	Quotient (or product) rule consistent with their derivs	$\dfrac{vu'-uv'}{v^2}$; allow one slip, missing brackets
			$=\dfrac{2\sin x\cos x+2\cos^2 x-2\sin x\cos x+2\sin^2 x}{(\sin x+\cos x)^2}$	A1	Correct expanded expression (could leave the '2' as a factor)	$\dfrac{uv'-vu'}{v^2}$ is M0. Condone $\cos x^2$, $\sin x^2$
			$=\dfrac{2\cos^2 x+2\sin^2 x}{(\sin x+\cos x)^2}=\dfrac{2(\cos^2 x+\sin^2 x)}{(\sin x+\cos x)^2}$			
			$=\dfrac{2}{(\sin x+\cos x)^2}$ *	A1	**NB AG**	must take out 2 as a factor or state $\sin^2 x+\cos^2 x=1$
			When $x=\pi/4$, $g'(\pi/4)=2/(1/\sqrt{2}+1/\sqrt{2})^2$	M1	substituting $\pi/4$ into correct deriv	
			$=1$	A1		
			$f'(x)=\sec^2 x$	M1	o.e., e.g. $1/\cos^2 x$	
			$f'(0)=\sec^2(0)=1$, [so gradient the same here]	A1		
				[7]		

Question			Answer	Marks	Guidance	
8	(iii)		$\int_0^{\pi/4} \mathrm{f}(x)\,\mathrm{d}x = \int_0^{\pi/4} \frac{\sin x}{\cos x}\,\mathrm{d}x$ let $u = \cos x$, $\mathrm{d}u = -\sin x\,\mathrm{d}x$ when $x = 0$, $u = 1$, when $x = \pi/4$, $u = 1/\sqrt{2}$			
			$= \int_1^{1/\sqrt{2}} -\frac{1}{u}\,\mathrm{d}u$	M1	substituting to get $\int -1/u\ (\mathrm{d}u)$	ignore limits here, condone no du but not dx allow $\int 1/u\ .-\mathrm{d}u$
			$= \int_{1/\sqrt{2}}^{1} \frac{1}{u}\,\mathrm{d}u$ *	A1	NB **AG**	but for A1 must deal correctly with the −ve sign by interchanging limits
			$= [\ln u]_{1/\sqrt{2}}^{1}$	M1	$[\ln u]$	
			$= \ln 1 - \ln(1/\sqrt{2})$ $= \ln\sqrt{2} = \ln 2^{1/2} = \frac{1}{2}\ln 2$	A1 **[4]**	$\ln\sqrt{2}$, $\frac{1}{2}\ln 2$ or $-\ln(1/\sqrt{2})$	mark final answer
8	(iv)		Area = area in part **(iii)** translated up 1 unit.	M1	soi from π/4 added	or
			So $= \frac{1}{2}\ln 2 + 1 \times \pi/4 = \frac{1}{2}\ln 2 + \pi/4$.	A1cao **[2]**	oe (as above)	$\int_{\pi/4}^{\pi/2} (1 + \tan(x - \pi/4))\,\mathrm{d}x = [x + \ln\sec(x - \pi/4)]_{\pi/4}^{\pi/2}$ $= \pi/2 + \ln\sqrt{2} - \pi/4 = \pi/4 + \ln\sqrt{2}$ B2
9	(i)		At P(a, a) g(a) = a so $\frac{1}{2}(\mathrm{e}^a - 1) = a$ $\Rightarrow \quad \mathrm{e}^a = 1 + 2a$ *	B1 **[1]**	**NB AG**	
9	(ii)		$A = \int_0^a \frac{1}{2}(\mathrm{e}^x - 1)\,\mathrm{d}x$	M1	correct integral and limits	limits can be implied from subsequent work
			$= \frac{1}{2}\left[\mathrm{e}^x - x\right]_0^a$	B1	integral of $\mathrm{e}^x - 1$ is $\mathrm{e}^x - x$	
			$= \frac{1}{2}(\mathrm{e}^a - a - \mathrm{e}^0)$	A1		
			$= \frac{1}{2}(1 + 2a - a - 1) = \frac{1}{2}a$ *	A1	**NB AG**	
			area of triangle $= \frac{1}{2}a^2$	B1		
			area between curve and line $= \frac{1}{2}a^2 - \frac{1}{2}a$	B1cao **[6]**	mark final answer	

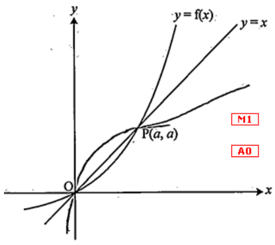
Question		Answer	Marks	Guidance	
9	(iii)	$y = ½(e^x – 1)$ swap x and y $x = ½(e^y − 1)$			
		$\Rightarrow$ $2x = e^y – 1$	M1	Attempt to invert – one valid step	merely swapping x and y is not 'one step'
		$\Rightarrow$ $2x + 1 = e^y$	A1		
		$\Rightarrow$ $\ln(2x + 1) = y$ *	A1	$y = \ln(2x + 1)$ or $g(x) = \ln(2x + 1)$ **AG**	apply a similar scheme if they start with $g(x)$ and invert to get $f(x)$.
		$\Rightarrow$ $g(x) = \ln(2x + 1)$			or $g\,f(x) = g((e^x – 1)/2)$ M1
		Sketch: recognisable attempt to reflect in $y = x$	M1	through O and (a, a)	$= \ln(1 + e^x − 1) = \ln(e^x)$ A1 $= x$ A1
		Good shape	A1	no obvious inflexion or TP, extends to third quadrant, without gradient becoming too negative	similar scheme for fg See appendix for examples
			[5]		
9	(iv)	$f'(x) = ½ e^x$	B1		
		$g'(x) = 2/(2x + 1)$	M1	$1/(2x + 1)$ (or $1/u$ with $u = 2x + 1$) …	
			A1	… × 2 to get $2/(2x + 1)$	
		$g'(a) = 2/(2a + 1)$, $f'(a) = ½ e^a$	B1	either $g'(a)$ or $f'(a)$ correct soi	
		so $g'(a) = 2/e^a$ or $f'(a) = ½(2a+1)$ $= 1/(½e^a)$ $= (2a + 1)/2$ $[= 1/f'(a)]$ $[= 1/g'(a)]$	M1 A1	substituting $e^a = 1 + 2a$ establishing $f'(a) = 1/g'(a)$	either way round
		tangents are reflections in $y = x$	B1	must mention tangents	
			[7]		

APPENDIX 1

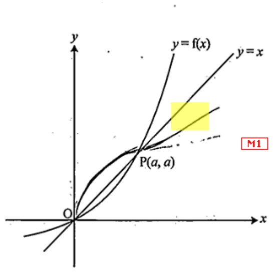
Exemplar marking of 9(iii)



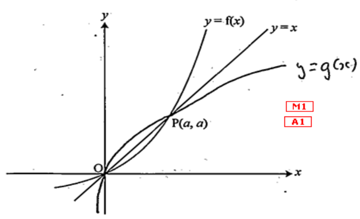
M1A1



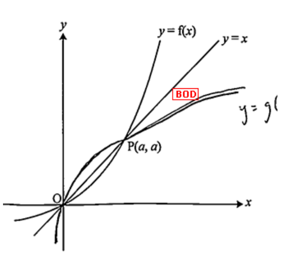
M1A0 – infl



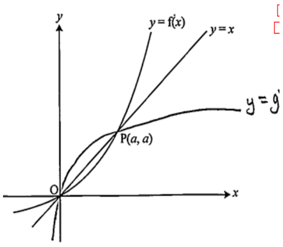
M1A0 –infl, no 3rd quadrant



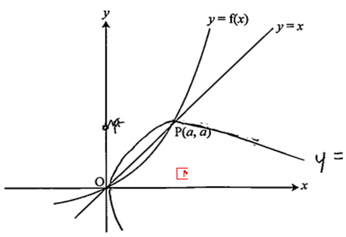
M1A1 (slight infl – condone)



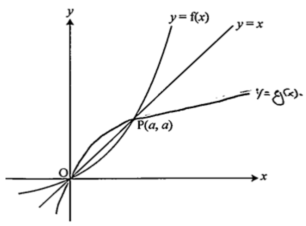
M1A1 (slight infl)



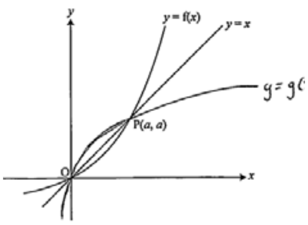
M1A0 (tp, just)



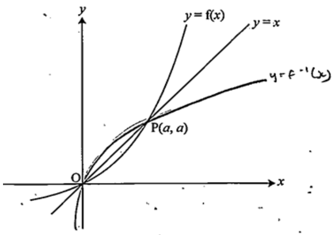
M1A0



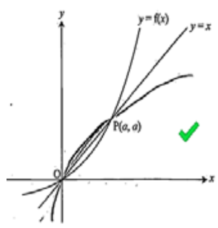
bod M1A1 (slight infl)



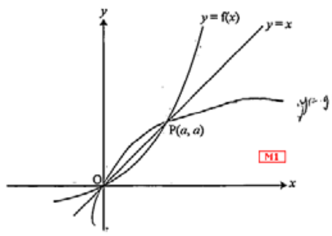
bod M1A1 (condone TP)



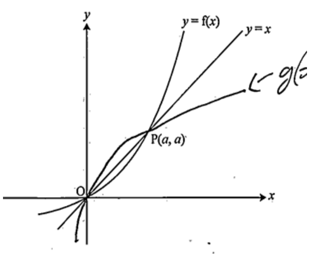
bod M1A1, (see 3rd quad)



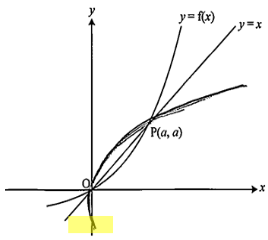
M1A1



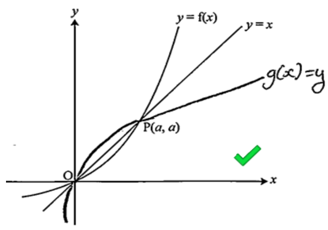
M1A0 (TP, 3rd quad)



M1A1



M1A0 (see 3rd q)



M1A0 (3rd q bends back)

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

January 2013

4753 Core C3

Over the years we have seen an increased use of software packages. This is to be encouraged, but it results in an increasing responsibility on the candidate to demonstrate an understanding of the method. For instance, it is possible to achieve the correct accuracy for the roots of an equation in domain 2 including a graph and production of tangents using some software packages. Such use does no more than demonstrate an understanding of how to use the package. It certainly does not indicate an understanding of the method. Candidates need to demonstrate an understanding by using the method, finding the derivative, choosing an initial value that is known to be “close to the root” (usually one of the end points of the integer interval within which the root lies) and showing how the iterates are produced.

Sometimes marks were given for hypothetical cases where the illustration does not match the arithmetical work. Other cases include comments such as “the method would fail if I tried on a graph like this” with no equation being given and no iterates shown.

Assessors are also reminded that the graph of an equation is not in itself an illustration of the method.

Domain 1

Occasionally we have observed that the root was quoted to an accuracy not justified by the working in the tables. It is expected that candidates will find, for instance $f(1.234) > 0$ and $f(1.235) < 0$ leading to the root being given as 1.2345 ± 0.0005. It is not correct to give, without further working, the root as either end of the interval. Please note also that a value for the root is expected; giving the interval [1.234, 1.235] is not a value for the root.

Domain 2

Roots are not always given to 5 significant figures, and the error bounds are often not justified by change of sign.

The illustration should include two clearly drawn tangents. Often the point at which the line is a tangent is not shown or the second tangent is not obvious – both of these can be resolved by a change in scale on the axes.

Domain 3

The most significant problem in this domain was a justification for convergence. It is acceptable (though not a requirement) for candidates to differentiate their function and to find its value close to the root (the initial value should not be used) and compare with the criterion for convergence. Alternatively a geometric argument may be employed, commenting on the gradient in comparison with that of the line $y = x$ (which has a gradient of 1) at the point of intersection. A significant number of candidates do not complete either method completely but are given credit.

Domain 4

The first criterion is clear – one of the equations should be used to find the same root that has already been found in one of the domains by the other two methods. We have seen credit given where a different root has been found or the same root to a different level of accuracy. Additionally, in order to compare speed of convergence it is necessary to start the iterative methods at the same initial value. Failure to do so renders any discussion of speed meaningless.

Candidates do not often state and discuss the value of the software they have used but are given credit. If they have used an Excel spreadsheet, for instance, then they should say so and describe how it has made the task easier.

Domain 5

The purpose of the task is to investigate the solution of equations. A significant number of candidates do not write any equations! We see many times a comment of the cover sheet saying "fine" and then turn to the first domain and see the candidate writing "I am going to solve the equation $x^3 + 3x – 5$." Candidates who persist in writing functions or expressions instead of equations, or refer to finding roots of a function should be penalised in this domain.

4758: Differential Equations

This unit is not entered by many candidates in the winter series so it is difficult to make many generalisations. However, one point worth making is that, where it is appropriate, a force diagram should be included when deriving the model. Its omission should affect the marking in Domain 1.

Whilst most students investigating 'Aeroplane Landing' did so correctly, scripts are still being received where the initial model does not take into account the braking force. That is, a decision is made to reject the initial model based on only the first 9 seconds. This should be penalised in Domains 2 and 3.

Finally, when modifying the initial model a justification must be given for the revised model. In particular, in many cases, there is a tendency to simply state that eg resistance is proportional to v^2 with no justification as to why that might be a possibility.

4776: Numerical Methods

The vast majority of candidates chose to do a numerical integration and so these comments refer only to this task, though some general points may be transferred to other tasks.

Domain 1

It is expected that candidates will state the investigation to be undertaken in precise mathematical terms. If an area is to be found numerically then a correctly written integral (with limits and "d*x*" included!) is required.

It is acceptable for candidates to assert that the integral they have chosen cannot be solved analytically. They do not have to "try" standard methods known to them which lead nowhere.

There are some integrals which are solvable analytically within the A level specification, but it is possible that candidates will have embarked on this further Mathematics Unit before completing the A level course. Such integrals are therefore acceptable. What is not acceptable is that candidates, having asserted that they cannot do the integral by analytical methods, then proceed to give such a solution. This leads them down a route that prevents them from gaining marks in that there is then a tendency to compare the values they obtain through the numerical process with a "known" value.

Domain 2

For numerical integration there are three standard methods. Any one of the three can be used to find a value to the required minimum accuracy. Therefore, if candidates are going to use more than one such method (or all three) then the reason for doing so (and there are many good reasons) should be stated. It is not necessary to describe how they work but why they are being used. This is a common error in this domain that is frequently rewarded.

Domain 3

For numerical integration it is expected that for a “substantial application”, candidates will work their chosen method(s) to at least 64 strips.

Domain 4

The technology used is usually an Excel spreadsheet; what is important for the second mark is that the formulae inserted into the cells are explained. This means more than just giving a printout showing what they are.

Domain 5

The problem of candidates who know the answer before they undertake the work has been mentioned above. If they then carry out some sort of analysis of differences between their values and the known values then the criteria of this domain have not been satisfied. The expectation is that the analysis of errors should be worked from within the process and conclusions achieved as a result of that analysis. Failure to do so means that the marks in this domain and in the next are not available and so credit should not be given.

Some candidates choose to find an integral on a function that is not well behaved. The most common problems are if $y = f(x)$ is not defined at an end point or the gradient is infinite at an end point. In these cases the ratio of differences will not converge in the way that theory suggests. Far from avoiding such cases however, a candidate would find such a case rich in opportunities to discuss validity and limitations. It is crucial, therefore, to establish the value to which the ratios converge and the work must be done to find them. Assuming the theoretical value is not credit worthy and may also lead to an incorrect solution.

Domain 6

The criterion for accuracy is not 6 significant figures. This is a minimum to be expected. The solution should be expressed to an accuracy that is justified in the work. Usually this will be more than 6 significant figures and part of the task is to be able to discern the level of accuracy that is valid from the error analysis.

Limitations could, as described above, relate to the value that is used for the ratio of differences to extrapolate to a best solution including the fact that rarely has the value subsequently used been achieved. It should be noted that the number of significant figures used by Excel means that the software will rarely limit the solution.

4753 Methods for Advanced Mathematics (Written Examination)

General Comments

This paper proved to be of an equivalent standard to recent C3 examinations. There were many excellent scripts, with over a quarter of the candidates scoring over 60 marks. There were very few really weak scripts, with less than 10% of candidates scoring fewer than 25 marks. This suggests than most candidates are well prepared for the examination. Few had time problems in completing the paper in the allotted time.

In general, the topics which were answered best on this paper were differentiation techniques, exponential growth and decay, and transformations of functions, Weaker topics were integration by parts and/or substitution, and calculus applied to ex and ln x functions.

The presentation of scripts was generally good. However, in more extended questions, such as 9(iv), the notion of presenting proofs in a coherent and logical manner often proved to be lacking, with candidates casting about and writing statements in random order and fashion. In particular, there is a tendency for candidates to 'argue backwards': for example, by starting from f'(a) = 1/g'(a), and arriving at ea = 1 + 2a! While we generally condone this, we hope that this practice is discouraged in the classroom.

Comments on Individual Questions

1(i) This proved to be a straightforward start to the paper, with the large majority of candidates getting full marks. Of those who did not, the most common errors were in the derivative of sin 2x (getting cos 2x or ½ cos 2x) or e−x (omitting the negative sign).

(ii) This part was somewhat less successful. Quite a few candidates just substituted the given answer into the derivative and claimed that this was zero.

2(i) This relatively simple implicit differentiation was very well done by almost all candidates.

(ii) Most candidates scored two out of three for the point (2, √2), but missed the y = −√2 solution. In a few cases, the denominator was set to zero, giving y = 0.

3 The non-standard nature of the question made this one of the harder section A questions. Some candidates were able to write the answer down while others used an algebraic approach.

4(i) In general, this is a well-known topic which is done successfully. Candidates who managed to deduce that a = 100 using $e^{-kt} \to 0$ as $t \to \infty$ usually gained full marks; those who did not often wasted time trying to solve simultaneous equations using $a - b = 15$ and $30 = a - b\,e^{-k}$.

(ii) There was an easy method mark to be gained from following through their values of *a, b* and *k*. Almost all who got these correct in part (i) scored both marks here, though very occasionally a premature rounding of k produced an insufficiently accurate answer.

5(i) This was almost invariably correctly done. No candidates seemed to be put off by the rather excessive speed of the car. Occasionally, the quotient rule was seen, with errors in differentiating the '25'.

(ii) Again, this was very well answered, provided part (i) was correct. Almost all candidates scored an M1 for the chain rule.

6 Most candidates used integration by substitution, though a significant minority used integration by parts. In general, the former were more successful, with the main difficulty being in expanding $(u – 1)u^{−1/2}$ as $u^{1/2} – u^{−1/2}$. Some proceeded from here using integration by parts, with mixed success. When parts were used, the most common error was in deriving $v = 2(1 + x)1/2$ from $v' = (1 + x)^{−1/2}$.

7(i) This proved to be very straightforward, with nearly everyone quoting the first correct counter-example of 245 (though a few came up with some much larger numbers).

(ii) This was not quite so easy as part (i). Most candidates who got full marks spotted the cyclic pattern in the units digits of 3^n as n increases. However, a significant minority evaluated 3^n for $n = 0$ to 9 and then cited 'proof by exhaustion'. The second approach, less commonly used, was to use the fact that numbers ending in '5' must be multiples of 5, and 3^n contains no factors of 5. However, many candidates who used this approach were unable to express the argument clearly enough and made incorrect statements.

8(i) We usually insist on the word 'translation' here, but in this case allowed 'move', 'shift', etc. A vector on its own does not in our view imply a translation. Occasionally, candidates clearly knew what the transformations were, but wrote the vectors incorrectly, for example the wrong way up. Nevertheless, this topic is usually well known and done well.

(ii) The quotient rule is generally well known, and errors here usually stemmed from faulty derivatives or poor algebra. Brackets are not optional in an expression like this, and their removal was not always successfully achieved. We also needed evidence of the use of $\cos^2 x + \sin^2 x = 1$, either by its direct quotation or by factoring out the '2' in the numerator. The evaluation of g'(x) was usually correct. With f'(x), some used a quotient rule on sin x/cos x rather than quoting the derivative of tan $x = \sec^2 x$; we also got some occasional 'translation' arguments here which misunderstood the nature of the verification.

(iii) This was a case where giving the transformed integral proved to be of doubtful value, as many candidates 'lost' the negative sign in their $\int -1/u \, du$, and placed the limits the wrong way round. It appears that the idea of swapping limits making the integral negative was not generally understood. The evaluation of the given integral with respect to u was more successfully done, though quite a few candidates approximated their final answer.

(iv) These marks were gained by candidates who managed to spot the rectangle of area added by the translation upwards of the graph of f(x).

9(i) This mark was usually earned.

(ii) Virtually everyone scored M1 for writing down the correct integral and limits, but many candidates made a meal of trying to integrate $\frac{1}{2}(e^x – 1)$, with $\frac{1}{4}(e^x – 1)2$ not an uncommon wrong answer. Having successfully negotiated this hurdle, using part (i) to derive ½ a was spotted by about 50% of the candidates. Quite a few candidates managed to recover to earn the final 2 marks for $\frac{1}{2}(a^2 – a)$ (without incorrectly simplifying this to ½ a!).

(iii) Finding the inverse function proved to be an easy 3 marks for most candidates – candidates are clearly well practiced in this. The graphs were usually recognisable reflections in $y = x$, but only well drawn examples – without unnecessary maxima or inflections – were awarded the 'A' mark.

(iv) This proved to be more difficult, as intended for the final question in the paper. As with the integral, many candidates struggled to differentiate ½ (e^x – 1) correctly, and equally many omitted the '2' in the numerator of the derivative of ln(1 + 2x). Once these were established correctly the substitution of $x = a$ and establishing of f'(a) = 1/g'(a) was generally done well, though sometimes the arguments using the result in part (i) were either inconclusive or done 'backwards'. The final mark proved to be elusive for most, as we needed the word 'tangent' used here to provide a geometric interpretation of the reciprocal gradients.